Spectral statistics of the two-body random ensemble revisited

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Using longer spectra we reanalyze spectral properties of the two-body random ensemble studied 30 years ago. At the center of the spectra the old results are largely confirmed, and we show that the nonergodicity is essentially due to the variance of the lowest moments of the spectra. The longer spectra allow us to test and reach the limits of validity of French's correction for the number variance. At the edge of the spectra we discuss the problems of unfolding in more detail. With a Gaussian unfolding of each spectrum the nearest-neighbor spacing distribution between ground state and first exited state is shown to be stable. Using such an unfolding the distribution tends toward a semi-Poisson distribution for longer spectra. For comparison with the nuclear table ensemble we could use such unfolding obtaining similar results as in the early papers, but an ensemble with realistic splitting gives reasonable results if we just normalize the spacings in accordance with the procedure used for the data.

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I. INTRODUCTION

The Gaussian orthogonal ensemble (GOE) was originally introduced by Wigner [1] in 1951 into physics in order to describe statistics of isolated, high-lying nuclear levels. A detailed analysis of each state seemed neither possible nor desirable. Statistical properties, on the other hand, were needed for nuclear technology and were well described by the GOE. About a quarter of a century ago there was considerable interest in the so-called embedded ensembles and in particular in the two-body random Hamiltonian ensemble (TBRE) [2–5], because it was realized that the GOE represented an n-body interaction in a mean-field basis, while it is generally assumed that nuclei can be fairly well described by an effective two-body interaction in this basis. Thus n-body Hamiltonians were constructed from two-body GOE's assuming degenerate single-particle states. The spectral statistics of these two-body random ensembles were analyzed. The main findings were

(1) The level density of the TBRE is Gaussian [2–4], rather than semicircular.

(2) The TBRE is neither stationary [5] nor ergodic [6,7].

(3) Unfolding the TBRE individually GOE statistics is recovered [6,7].

(4) The deviation with respect to GOE is due to the variance of average spacings over the ensemble [6].

(5) Fluctuations at the edge of the spectrum of a TBRE are very large [5] compared to those of a GOE and can be roughly described by a Brody distribution [8] of nearest-neighbor spacings with a parameter $\omega = 0.85$ [9].

These results, with the exception of the one for the level density, are exclusively numerical. They were obtained at a time of very limited computer facilities. As a consequence, only few particles were treated and the dimension of the TBRE matrices was small.

There has been recently a flurry of interest in the TBRE regarding properties of wave functions and spectra [10– 22]. In this context the old work was ignored to a large extent. Yet it seems worthwhile to take it into account, because it is not clear what effects nonergodicity could have in the new context. The numerical results can be improved considerably upon and by consequence the old results can be tested. Furthermore it seems desirable to go beyond the TBRE to what we may call a realistic two-body random ensemble (RTBRE) by lifting the degeneracy condition on the single-particle spectrum. Applications in nuclear physics and in other fields certainly require this.

In what follows we will first fix notation defining the TBRE properly and proceed in Sec. III to analyze properties in the center of the spectrum. In Sec. IV we consider properties at the edge of the spectrum and we discuss the RTBRE which seems to differ mainly there. Conclusions will be given in the last section.

II. DEFINITION OF THE TBRE

The TBRE is defined for a fermionic system of n spin-1/2 particles in three dimensions. A set of degenerate single-

TABLE I. Quantum numbers, dimension of the matrices, and the number of members for the ensembles analyzed in this paper.

	(J,T)	Dimension	Number of matrices
TBRE	(0,0)	325	500
TBRE	(0,2)	287	500
TBRE	(2,0)	1206	708
RTBRE	(0,0)	325	500

particle states with well-defined angular momentum and other quantum numbers is used to construct *n*-particle states, that belong to a good total angular momentum *J*, and if we think of a nucleus, also a good isospin *T*. Due to the Pauli principle and the corresponding blocking of states, the lowest single-particle states may not enter the picture because we assume them to be filled and inert. The effective two-body interaction usually used in this context was replaced by a Gaussian distributed two-body interaction matrix whose strength is irrelevant as no energy scale is established by degenerate single-particle states.

For our numerical studies we choose a strength parameter λ such that the value $\lambda=1$ corresponds to a typical interaction strength in the nuclear $2s-1d$ shell on which we shall concentrate. This parameter will be important for the RTBRE, where realistic mean-field parameters yield the relevant single-particle energies and thus provide an energy scale. The calculations reported here were mainly carried out for eight particles in the $2s-1d$ shell. Information about the quantum numbers, dimension of the spectra, and the number of matrices of each ensemble used can be found in Table I. The corresponding shell-model calculations were carried out using the code OXBASH [23]. As far as spectral statistics are concerned we shall mainly consider the nearest-neighbor spacing distribution $P(s)$ and the number variance $\Sigma^2(L)$ and occasionally the skewness $\gamma_1(L)$ and the excess $\gamma_2(L)$ [24]; all refer to unfolded spectra, i.e., to spectra whose local average density is normalized to one. The unfolding has some fine points which will be essential to this paper and will be discussed in the next section. We recall the definitions of the spectral fluctuation measures: $P(s)$ gives the probability to find two energy levels at distance *s*. $\Sigma^2(L)$ is the variance of the number of levels $N(L)$ in an energy interval of length L and

$$\gamma_1(L)=\frac{\Sigma^3(L)}{(\Sigma^2(L))^{2/3}} \quad \text{and} \quad \gamma_2(L)=\frac{\Sigma^4(L)}{(\Sigma^2(L))^2}-3$$

are combinations of higher moments $\Sigma^i=[N(L)-L]^i$. As we assume stationary spectra the results do not depend on the initial point of the interval L, and indeed usually averages will be taken over both ensembles and some energy window. This will be specified in each case.

III. STATISTICS IN THE CENTER OF THE TBRE AND NONERGODICITY

Before we proceed to analyze statistics of spectral fluctuations we have to unfold the spectrum [24] in order to have average spacing $D=1$ throughout the spectrum. For this purpose we could use the analytic result that we have a Gaussian density for the TBRE [4]. For practical purposes it turns out to be convenient simply to make a polynomial unfolding, using a best fit to the cumulative level density of the ensemble. For the central region of the spectrum the type of unfolding used is indeed not relevant.

We thus superpose all spectra and the integrated level density (staircase) is fitted with a polynomial of degree 15. Each spectrum is unfolded with this polynomial for an energy interval at the center of the spectra which contains on the average 115 states and calculated $P(s)$ and $\Sigma^2(L)$. The results are shown in Figs. 1(a) and 1(b). The short-range fluctuations deviate significantly from the GOE and the long-range correlations show huge deviations from this ensemble. This becomes rather obvious if we look at the distribution of the average energy and width of each spectrum individually. In Table II we compare the following variances: σ_c of the distribution of the average energies of each spectrum, σ_v of the widths distribution of each spectrum and σ_{tot}, which is the width of the energy distribution of the entire ensemble. The scattering of the centers and the widths of the TBRE and the RTBRE in units of the total widths is about 30 times larger than the one of the GOE. Both quantities fluctuate

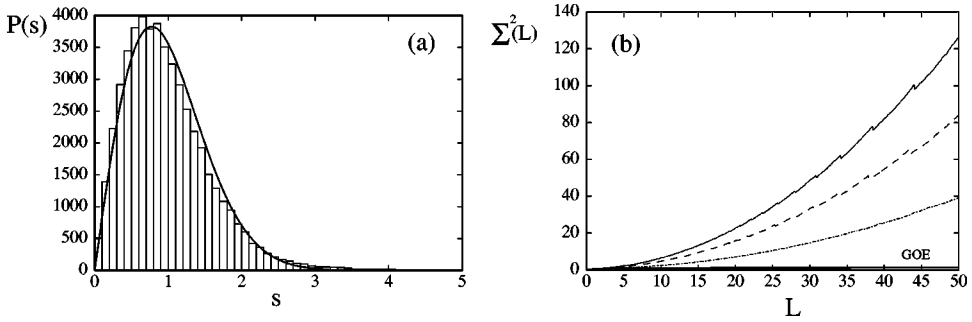

FIG. 1. (a) Ensemble averaged nearest-neighbor spacing distribution of the TBRE ($J=0$, $T=0$) at the center of the spectrum. The full line corresponds to the GOE; the ordinate is normalized to show the number of spacings in each bin of the histogram, and the abscissa to the average spacing 1. (b) Number variance $\Sigma^2(L)$ for the same case. The thin solid line corresponds to the ensemble average, the dashed line to the ensemble average after normalizing the spectrum widths, and the dashed-dotted curve to the ensemble average after recentering the spectra. The GOE values are indicated by the thick solid line. The normalization of the average level spacing defines the scales.

TABLE II. Standard deviation of the center distribution σ_c, the distribution of the widths of the spectra σ_v, and of the total level density of the ensemble σ_{tot} for three different TBRE's and the GOE.

	σ_c	σ_v	σ_{tot}	σ_c/σ_{tot}	σ_v/σ_{tot}
TBRE $J=0,\ T=0$	20.1	4.5	39.7	0.51	0.11
TBRE $J=0,\ T=2$	22.1	4.3	37.1	0.59	0.12
TBRE $J=2,\ T=0$	20.3	4.2	38.2	0.53	0.11
RTBRE $J=0,\ T=0$	5.0	1.96	12.3	0.41	0.16
GOE	0.015	3.17×10^{-3}	1.0	1.5×10^{-2}	3.1×10^{-3}

widely while for the GOE the distributions are extremely narrow.

This seems to indicate that the TBRE is not ergodic, and thus unfolding with the ensemble averaged level density is inappropriate. This is in contradiction with the findings of French [6], where he shows that the spectral width fluctuations (identical to his density fluctuations) decrease with $1/\kappa$, where $\kappa^2/2$ is the number of independent matrix elements in a one-dimensional (symmetry free) TBRE. This result has recently been confirmed [25] and holds similarly for the fluctuations of the positions of the centers of the spectra. There is good reason to believe that it also holds in the shell-model though the compexity of the relations involving angular momentum, spin and isospin preclude an analytic proof. Nevertheless the statement that rapid convergence may be expected as we increase the number of single-particle levels (responsible for the independent matrix elements) [6] is deceiving. For any practical situation the total size of the Hilbert space is relevant, both because of the numerical diagonalization aspect and because the number of truely participating states is always limited, though possibly big. Yet if we assume a constant filling factor of the shell, i.e., that the particle number increases as the number of single-partice states, the fluctuations decrease essentially like $1/\ln(N)$, and for finite particle number m like $1/N^{2/m}$. This makes for very poor convergence. For the case with symmetry, the ratio of independent matrix elements to number of states is in general even less favorable, and thus for practical purposes we cannot assume ergodicity except if we truely can take the limit analytically.

We, therefore, proceed to adjust a polynomial of degree 7 to each spectrum individually and use this individual density for the unfolding procedure.

Figure 2(a) and 2(b) show that now $P(s)$ and $\Sigma^2(L)$ are in good agreement with the GOE prediction and Figs. 3(a) and 3(b) show a similar agreement for the skewness and excess. We should remark that in this case we could increase the energy interval and decrease the degree of the polynomial in order to get the high quality of the result for $\Sigma^2(L)$ up to $L=50$. We discarded 85 states both at the upper and lower end of the spectra. These numbers were chosen because omitting fewer states showed significant edge effects while omitting more reduced the range of L for which the fluctuations of $\Sigma^2(L)$ were acceptable.

We next wish to check whether the variations of the centers and widths of the spectra were alone responsible for the deviations with respect to the GOE. Therefore, we repeated the calculations with an ensemble unfolding after first recentering all spectra to the same value or, alternatively, normalizing their width to the same value. The result for the number variance for both procedures is drawn in Fig. 1(b). Both curves for the number variance represent an improvement but remain very far from GOE. Obviously we should apply both corrections, i.e., both recenter and dilate the spectra such as to correspond to uniform centers and widths before the ensemble unfolding. The result is such that on the scale of Fig. 1(b) it is not distinguishable from the GOE. The result is therefore plotted in Fig. 2(b) and is comparable to the one obtained by unfolding the individual spectra.

On the average we discarded 105 states at the edges of the

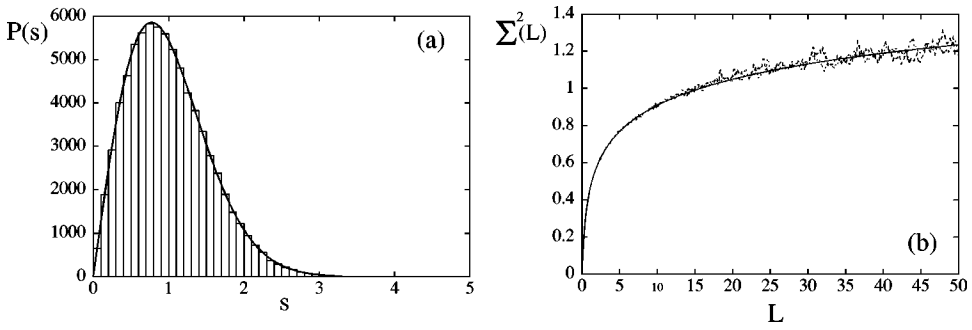

FIG. 2. (a) Nearest-neighbor spacing distribution of the TBRE ($J=0,\ T=0$) after each spectrum is unfolded individually. The solid line corresponds to the GOE. (b) Number variance $\Sigma^2(L)$ for the same case. Scales as in Fig. 1. The dotted and dashed curves correspond, respectively, to spectral average and ensemble average after recentering the spectra and normalizing their widths. The full line is obtained for the GOE.

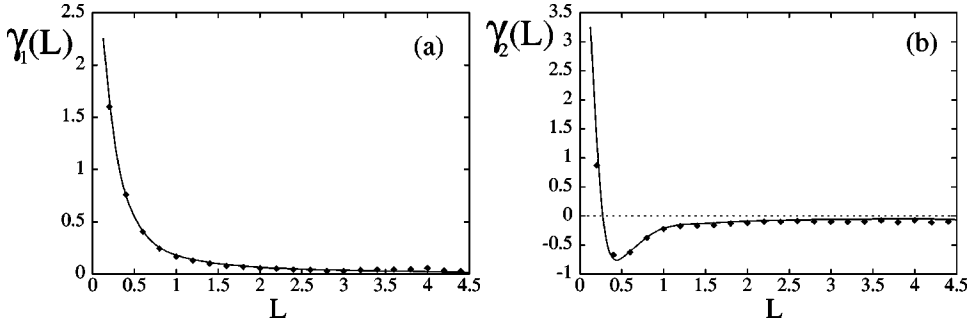

FIG. 3. (a) Skewness $\gamma_1(L)$ and (b) excess $\gamma_2(L)$ for the spectral average of the TBRE. The full line corresponds to the GOE. The scales are determined by the average level spacing 1.

spectra and found $m=15$ as the optimum degree for the polynomial in this analysis. The quality of the energy averaged $\Sigma^2(L)$ is only marginally better. Higher moments play a minor role. We obtained the same result for an ensemble of comparable size and quantum numbers $J=0$, $T=2$ and $J=2$, $T=0$. Similar agreement was found for $P(s)$, as well as for skewness and excess.

These results seem to indicate that the first and second moments are basically responsible for the nonergodicity of the TBRE, but we have no theoretical argument to support this conjecture, and thus it might not always be true, or break down with higher exactitude.

The above procedure eliminates the large fluctuations in the average density of each spectrum, but it is not totally equivalent to the procedure proposed by French [6]. In the original paper the correction he calculated was given for the distribution of widths of nth neighbor spacings as a function of L. We translated the correction to one for $\Sigma^2(L)$ [see Eqs. (5.3) and (6.3) of Ref. [7]] that reads as

$$\Sigma_s^2(L)=\frac{\Sigma_e^2(L)+\left[\frac{1}{6}-L^2\right]\frac{\sigma^2}{D^2}}{1-\frac{\sigma^2}{D^2}}, \qquad (1)$$

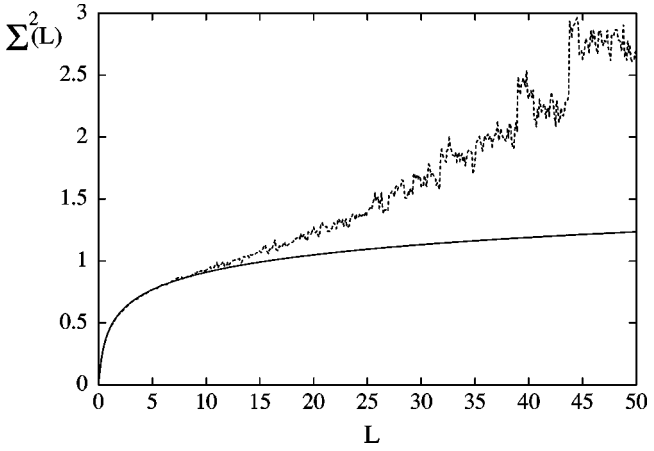

FIG. 4. Application of formula (1) as described in the text (dashed line) in comparison with the GOE (full line). Scales as in Fig. 1.

where $\Sigma_e^2(L)$ and $\Sigma_s^2(L)$ denote the ensemble and spectral (energy) averaged number variance at distance L and σ^2 denotes the ensemble variance of the spectrally averaged mean level distance D. Figure 4 shows $\Sigma_s^2(L)$ as calculated from Eq. (1) using the numerical data for $\Sigma_e^2(L)$. Note that this correction is applied after recentering the spectra. This is not explicit in the original paper, but it is the only reasonable way to interpret the basic argument given there. Indeed, application of the formula to nonrecentered spectra gives unreasonable results.

For correlations up to about $L=5$ and certainly for the nearest-neighbor spacing distribution the correction works quite well; at larger distances discrepancies become large, as is to be expected if the aim is to simulate the adjustment of the width of a near Gaussian. The larger the spectra the larger the range on which we expect the correction to work well. In earlier calculations with smaller ensembles the number variance was not considered at large distances, and thus the deviation could not be detected.

The spectral properties of the RTBRE at the center are also found to be of the GOE type up to some energy range which depends on the subspace defined by the quantum numbers we consider and on the interaction strength.

IV. PROPERTIES AT THE EDGE OF THE SPECTRA

The fluctuations at the edge of the spectra are of interest because of the comparison with low-lying nuclear states [9], and in the context of recent work on the angular momentum dependence of ground states [18,20,21]. The quantity to consider is the nearest-neighbor spacing distribution. The old results are fitted with Brody distributions $P(s,\omega)=\alpha(\omega+1)s^{\omega}\exp\{-\alpha s^{\omega+1}\}$ with $\alpha(s,\omega)=[\Gamma((\omega+2)/(\omega+1))]^{\omega+1}$ [8] and parameters around $\omega=0.5$. Recent studies have suggested the use of semi-Poisson distributions $P(s)=4s\exp\{-2s\}$ [26], that might be relevant as we are interested in the edge of the spectra. These distributions have been discussed in relation with triangles with irrational relations between angles [27], where the coincidence is spotty. We keep this additional option in mind to expand on the old discussions.

The simplest way to proceed is to take all spacings between the first and the second level and normalize them to

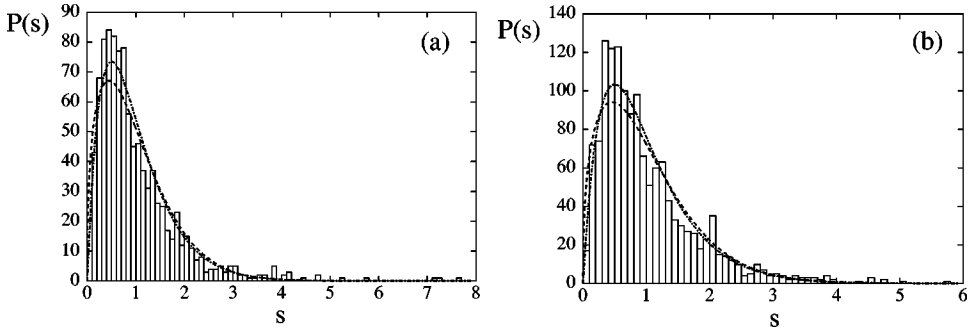

FIG. 5. (a) Distribution of the spacings between ground and first excited states for the TBRE ($J=0,\ T=0$). The dashed-dotted line corresponds to the semi-Poisson and the dashed line to the best Brody fit. (b) The same as in (a) for the quantum numbers ($J=2,\ T=0$). Scales as in Fig. 1.

average 1 (for improved statistics we use the corresponding spacings at the upper end of the spectrum also, without mentioning it explicitly each time). In Fig. 5 we show the spacing distribution obtained in this way, and we find amazingly large spacings up to 8 in the case $J=0,\ T=0$ and up to 6 for the ensemble with quantum numbers $J=2,\ T=0$. Note that the same procedure performed on a GOE of the same size leads to a spacing distribution very similar to that at the center of the spectrum. The semi-Poisson distribution does not yield a very convincing fit, but is not worse than the usual Brody distribution, despite the fact that the latter has one free parameter.

Normalizing the distances to 1 might seem too naive. Although the polynomial unfolding of the integrated spectrum delivered good results for the analysis of the central region it is not useful at the edges. Therefore, we proceed to use a Gaussian to unfold each spectrum individually adjusting its parameters. This procedure has some merit; a Gaussian is the exact result for the ensemble with infinite dimension, though the fact that we have already re-confirmed the nonergodicity of the TBRE casts some doubt on its application to individual spectra. In Fig. 6 we show the spacing distribution between the first and the second states for $J=0,\ T=0$ and $J=2,\ T=0$ unfolded in this way. The extremely large spacings have disappeared though the result still differs markedly from a GOE. The confidence levels, discussed in [8] for a Brody fit, rely heavily on binning the small intensities in the tail in one large bin and are therefore somewhat arbitrary. In particular, a few large spacings would not invalidate the fit. The analysis of the ensemble for $J=0,\ T=2$ delivered qualitatively the same result and they agree substantially with the old results. The semi-Poisson distribution drawn in the same picture gives a surprisingly good fit in Fig. 6(b) and is somewhat worse in Fig. 6(a). Note though that the discrepancies again are considerably smaller for the longer spectrum and are not larger for the shorter one than for the Brody distribution with an adjusted parameter.

Yet we may ask if the Gaussian unfolding is really that meaningful. In this context two questions arise: First, is the procedure stable? And second, does it relate to the only experimental data we have? While the latter point will be discussed later, the former may be considered by using a Gaussian multiplied by a polynomial for unfolding. We may then ask if we find any plateau of the result as a function of the degree of the polynomial. We considered the Brody parameter as a function of the degree of the polynomial and we find that for the short spectrum a polynomial up to degree 2 and for the longer one a polynomial up to degree 4 can be added without a significant change of the Brody parameter. This, combined with the theoretical expectation of a Gaussian in the limit of large matrices, gives us some confidence to use the unfolding mentioned and should encourage further analysis as to the significance of the agreement found with a semi-Poisson distribution.

We next look at the comparison with the nuclear physics results of Ref. [9]. Note that here the normalization of the spacings is made as a function of mass number and angular momentum, but in no way is the spectral density further up in the spectrum involved. We therefore conclude that only a comparison with the simply normalized spacings shown in

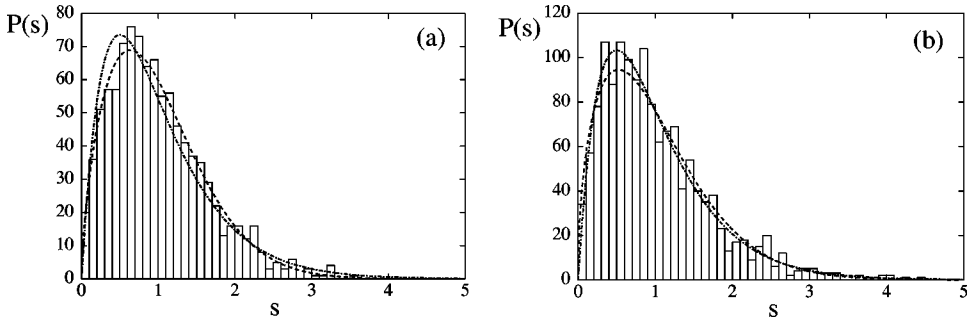

FIG. 6. The same as in Fig. 5, but unfolding each spectrum individually with a Gaussian.

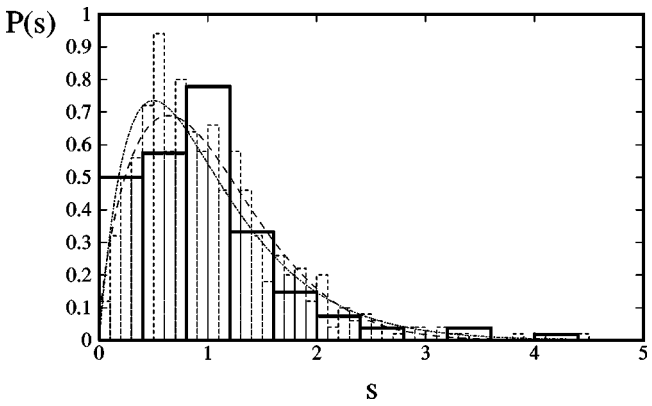

FIG. 7. Distribution of the spacing between ground and first excited states. Here the vertical scale gives the probability to find two levels at the distance *s*. The thick solid line histogram corresponds to the nuclear table ensemble and the thin solid line one to the RTBRE ($J=0,\ T=0$). The dashed-dotted line corresponds to the semi-Poisson and the dashed curve to the best Brody fit.

Fig. 5 is legitimate. Yet these data have a very long tail that seems incompatible with the nuclear data. As we discarded a more sophisticated unfolding, we have to fall back on an alternative option. Obviously the assumption of degenerate single-particle levels is not fulfilled in the nuclear case. It, therefore, seems justified to consider realistic splitting of the single-particle levels combined with a realistic interaction strength, i.e., the RTBRE. In this case we must limit ourselves to the spacings at the bottom of the spectrum, as symmetry is destroyed by the single-particle energies. Note that the results we now obtain are dependent on the shell splitting and by no means universal. Yet if we compare these results in Fig. 7 with the experimental ones we see less discrepancies. As the nuclear data have poor statistics (135 events) the agreement is not very meaningful, but at least the few large spacings are consistent and the general shape seems correct. Fits with a Brody distribution fails in this case and the agreement with a semi-Poisson distribution is similarly poor. Comparison with the unfolded TBRE for the smaller sample, on the other hand, would give a better fit, but has no real foundation.

V. CONCLUSIONS

We may conclude that the phenomenology observed in earlier papers for the TBRE is essentially correct when viewed with larger spectra. The correction formula proposed in [6] will only correct short-range behavior if we recenter the spectra previously. We found that recentering spectra and normalizing their widths would remove the nonergodicity at the center of the spectrum to a large extent. Note though, that this nonergodicity is not really a problem at least if we consider the center of the spectra, because in practice we have to unfold each nuclear, atomic, resonance cavity or whatever spectrum individually, anyway. At the center of the spectrum the fluctuation properties of the RTBRE coincide with those of the TBRE on the range determined by the strength of the interaction.

As far as edge effects are concerned, the situation changes. Spectral properties with realistic splitting of single-particle levels differ significantly from those of the TBRE. The RTBRE seems to be in better agreement with the nuclear data, but no doubt a new compilation of such data is desirable because the small sample does not allow a detailed comparison. The unfolding procedures at the edge of the spectrum are quite sensitive, but we have found unfolding with a Gaussian to be quite satisfactory both because of its theoretical background and because of its stability. We certainly confirm earlier findings of large fluctuations, and with the Gaussian unfolding we found very reasonable agreement with a semi-Poisson distribution for the longest spectrum we have.

We may conclude that future attention should be concentrated on two points: First we should look out for possible effects of the nonergodicity on wave-function properties, and second we should investigate the possible significance of the semi-Poisson distribution at the edge of spectra.

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